

Characterization of Dom-Set Chromatic Number of a Graph

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ABSTRACT: Let G be a graph. For a non-trivial connected graph G , let $C: V(G) \rightarrow \mathbb{N}$ be a vertex coloring of G where adjacent vertices may be assigned the same color. For a dom-coloring set $S \subseteq V(G)$, define $C(S)$ of colors assigned to the vertices of S by $C(S) = \{C(v): v \in S\}$. For a vertex v in a graph G , let $N(v)$ be the neighbourhood of v (the dom-coloring set of all vertices adjacent to v in G). The neighbourhood color set $NC(v) = C(N(v))$ is the set of colors of the neighbours of v . The new coloring c is called dom-set coloring if $NC(u) \neq NC(v)$ for every pair u, v of adjacent vertices of G . The minimum number of colors required of such a coloring is called the dom-set chromatic number of G and denoted by $\gamma_{(d-c)}(G)$. In this paper, we explain some characterization of dom-set chromatic number of graphs.

KEYWORDS: Chromatic number, dominating set, Domination number, Dom-set colouring set, Dom-set chromatic number

I. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let $G = (V, E)$ be a graph. The open neighborhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v . The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. An induced subgraph $G[S]$, where S of a graph G is a graph formed from a subset S of the vertices of G and all of the edges connecting pairs of vertices in S . A graph in which every pair of vertices is joined by exactly one edge is called complete graph. A complete bipartite graph is a graph whose vertices can be partitioned into two subsets V_1 and V_2 such that no edge has both end points in the same subset, and each vertex of V_1 is connected to every vertex of V_2 and vice versa. A star graph S_n is the complete bipartite graph $K_{1,n-1}$ (A tree with one internal node and $n-1$ leaves).

The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H , we define the cartesian product, denoted by $G \times H$, to be the graph with vertex set $V(G) \times V(H)$ and edges between two vertices (u_1, v_1) and (u_2, v_2) iff either $u_1 = u_2$ and $v_1 v_2 \in E(H)$ or $u_1 u_2 \in E(G)$ and $v_1 = v_2$.

A subset S of V is called a dominating set if every vertex in $V-S$ is adjacent to at least one vertex in S . The dominating set is minimal dominating set if no proper subset of S is a dominating set of G . The domination number γ is the minimum cardinality taken over all minimal dominating set of G . A γ -set is any minimal dominating set with cardinality γ .

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A dominator coloring of G is a proper coloring of G in which every vertex of G dominates at least one color class. The dominator chromatic number is denoted by $\chi_d(G)$ and is defined by the minimum number of colors needed in a dominator coloring of G . This concept was introduced by Raluca Michelle Gera in 2006[2].

In a proper coloring C of G , a color class of C is a set consisting of all those vertices assigned the same color. Let C' be a minimal dominator coloring of G . We say that a color class $c_i \in C'$ is called a non-dominated color class ($n-d$ color class) if it is not dominated by any vertex of G . These color classes are also called repeated color classes. For a non-trivial connected graph G , let $C: V(G) \rightarrow \mathbb{N}$ be a vertex coloring of G where adjacent vertices may be assigned the same color. For a dom-coloring set $S \subseteq (G)$, define $C(S)$ of colors assigned to the vertices of S by $C(S) = \{C(v): v \in S\}$. For a vertex v in a graph G , let $N(v)$ be the neighbourhood of v (the dom-coloring set of all vertices adjacent to v in

G). The neighbourhood color set $NC(v) = C(N(v))$ is the set of colors of the neighbours of v . The new coloring c is called dom-set coloring if $NC(u) \neq NC(v)$ for every pair u, v of adjacent vertices of G . The minimum number of colors required of such a coloring is called the dom-set chromatic number of G and denoted by $\gamma_{(d-c)_S}(G)$. For a graph G with dom-chromatic number k , let c be a proper k -dom-coloring of G . Suppose that u and v are adjacent vertices of G . Since $(u) \in NC(v)$ and $c(u) \notin NC(u)$ it follows that $NC(u) \neq NC(v)$. Hence every proper k -dom-coloring of G is also called set k -dom-coloring of G . Therefore, for every graph G , $\gamma_{(d-c)_S}(G) \leq \gamma_{d-c}(G)$.

II. MAIN RESULTS

THEOREM 1:

For every graph G , $\gamma_{(d-c)_S}(G) \geq \log_2[\gamma_{d-c}(G) + 1]$

Proof:

This is true if $1 \leq \gamma_{d-c}(G) \leq 3$.

We assume that $\gamma_{d-c} \geq 4$. Let $\gamma_{(d-c)_S} = k$ and let there be given a set k -dom coloring of G using the colors in \mathbb{N}_k . Thus $NC(x) \subseteq \mathbb{N}_k$ for every vertex x of G . Since $NC(u) \neq NC(v)$ for every two adjacent vertices u and v , it follows that $NC(x)$ can be considered as a color for each $x \in (G)$. i.e) The coloring c of G defined by $c(x) = NC(x)$ for $x \in V(G)$ is a proper coloring of G . Since there are $2^k - 1$ non empty subsets of \mathbb{N}_k , it follows that c uses at most $2^k - 1$ colors. Thus $\gamma_{d-c} \leq 2^k - 1$ which implies $\gamma_{d-c} + 1 \leq 2^k$ and taking log base 2 on both sides we get $\log_2[\gamma_{d-c}(G) + 1] \leq \log_2(2^k) = k$. Therefore, $\log_2[\gamma_{d-c}(G) + 1] \leq \gamma_{(d-c)_S}(G)$.

THEOREM 2:

For every graph G , $\gamma_{(d-c)_S}(G) \geq 1 + \lceil \log_2 \omega(G) \rceil$.

Proof:

If $\omega(G) = 2$, then $\gamma_{(d-c)_S}(G) \geq 2$, while if $\omega(G) = 3$ then $\gamma_{(d-c)_S}(G) \geq 3$. Thus we may assume that $\omega(G) = \omega \geq 4$.

Let H be a clique of order ω in G with $(H) = \{v_1, v_2, \dots, v_\omega\}$. Suppose that $\gamma_{(d-c)_S}(G) = k$ and let $c: V(G) \rightarrow \mathbb{N}_k$ be a set k -dom coloring of G . We consider 2 cases, according to whether there are two vertices $V(H)$ colored the same or no two vertices in $V(H)$ are assigned the same color.

Case1:

There are 2 vertices in $V(H)$ colored the same, say $c(v_1) = c(v_2) = 1$. Then $1 \in NC(v_i)$ for $1 \leq i \leq \omega$.

Since there are exactly 2^{k-1} subsets of \mathbb{N}_k containing 1, it follows that $\omega \leq 2^{k-1}$ and so

$k - 1 \geq \log_2 \omega$. This implies $k \geq \log_2 \omega + 1$. Therefore, $\gamma_{(d-c)_S}(G) \geq 1 + \lceil \log_2 \omega(G) \rceil$.

Case2:

No two vertices in $V(H)$ are colored the same. Then ω distinct colors are used for the vertices in $V(H)$ so $\omega \leq k$. Since $\omega \geq 4$, it follows that $k \geq \omega > 1 + \lceil \log_2 \omega(G) \rceil$.

Therefore, $\gamma_{(d-c)_S}(G) \geq 1 + \lceil \log_2 \omega(G) \rceil$.

THEOREM 3:

If v is a vertex of a graph G , then $\gamma_{(d-c)_S}(G) - 1 \leq \gamma_{(d-c)_S}(G - v) \leq \gamma_{(d-c)_S}(G) + \deg v$.

Proof:

First we verify the lower bound for $\gamma_{(d-c)_S}(G - v)$. Suppose that $\gamma_{(d-c)_S}(G - v) = k$. Let

$c_1: (G - v) \rightarrow \mathbb{N}_k$ be a set k -dom coloring of $G - v$. Then the coloring c' of G is defined by, c'_1

$$c' = \begin{cases} c_1(x) & \text{if } x \neq v \\ 1 & \end{cases}$$

$$k + 1 \text{ if } x = v$$

is a set coloring of G using $k + 1$ colors. Therefore $\gamma_{(d-c)_S}(G) \leq k + 1 = \gamma_{(d-c)_S}(G - v) + 1$. This implies that $\gamma_{(d-c)_S}(G) - 1 \leq \gamma_{(d-c)_S}(G - v) \rightarrow (1)$

Next we verify the upper bound for $\gamma_{(d-c)_S}(G - v)$.

Suppose that $\gamma_{(d-c)S}(G) = l$ and $\deg v = d$ where $N(v) = \{v_1, v_2, \dots, v_d\}$. Let $c_2: V(G) \rightarrow \mathbb{N}_l$ be the set l -dom coloring of G . Then the coloring c' of $G - v$ defined by

$$c'(x) = \begin{cases} c_2(x) & \text{if } x \notin N(v) \\ l+i & \text{if } x = v (1 \leq i \leq d) \end{cases}$$

is a set coloring of $G - v$, using at most $l + d$ colors. Therefore $\gamma_{(d-c)S}(G - v) \leq l + d = \gamma_{(d-c)S}(G) + \deg v$.

This implies that $\gamma_{(d-c)S}(G - v) \leq \gamma_{(d-c)S}(G) + \deg v \rightarrow (2)$

From (1) and (2),

$$\gamma_{(d-c)S}(G) - 1 \leq \gamma_{(d-c)S}(G - v) \leq \gamma_{(d-c)S}(G) + \deg v$$

THEOREM 4:

For any graph G and any edge $e \in (G)$, $|\gamma_{(d-c)S}(G) - \gamma_{(d-c)S}(G - e)| \leq 2$. Proof:

Let $e = uv$. First we show that $\gamma_{(d-c)S}(G) \leq \gamma_{(d-c)S}(G - e) + 2$.

Assume $\gamma_{(d-c)S}(G - e) = k$ and let $c_1: V(G - e) \rightarrow \mathbb{N}_k$ be a set k -dom coloring of $G - e$. Then observe that the coloring c' of G defined by

$$c'(x) = \begin{cases} k+1 & \text{if } x = u \\ k+2 & \text{if } x = v \\ c_1(x) & \text{otherwise} \end{cases}$$

is a set coloring of G that using at most $k + 2$ colors. Therefore $\gamma_{(d-c)S}(G) \leq k + 2 = \gamma_{(d-c)S}(G - e) + 2$. This implies that $\gamma_{(d-c)S}(G) - \gamma_{(d-c)S}(G - e) \leq 2 \rightarrow (1)$

Next, we show that $\gamma_{(d-c)S}(G - e) \leq \gamma_{(d-c)S}(G) + 2$. Suppose $\gamma_{(d-c)S}(G) = l$ and consider a set l -dom coloring $c_2: V(G) \rightarrow \mathbb{N}_l$ of G . Then the coloring c' defined by

$$c'(x) = \begin{cases} l+1 & \text{if } x = u \\ l+2 & \text{if } x = v \\ c_2(x) & \text{otherwise} \end{cases}$$

is a set coloring of $G - e$ using at most $l + 2$ colors.

Thus $\gamma_{(d-c)S}(G - e) \leq l + 2 = \gamma_{(d-c)S}(G) + 2$. This implies $\gamma_{(d-c)S}(G - e) \leq \gamma_{(d-c)S}(G) + 2$. Therefore, $\gamma_{(d-c)S}(G - e) - \gamma_{(d-c)S}(G) \leq 2 \rightarrow (2)$

From (1) and (2), $|\gamma_{(d-c)S}(G) - \gamma_{(d-c)S}(G - e)| \leq 2$

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